

CINVESTAV-FIS-16-95

T-violation in $K_{\mu 3}$ decay in a general two-Higgs doublet model.

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Abstract

We calculate the transverse muon polarization in the $K_{\mu 3}^+$ process arising from the Yukawa couplings of charged Higgs boson in a general two-Higgs doublet model where spontaneous violation of CP is present.

PACS numbers: 11.30.Er, 12.60.Fr, 13.20.Eb, 14.80.Cp,

It has been suggested [1] that the muon polarization transverse (P_μ^\perp) to the decay plane in the process $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ ($K_{\mu 3}^+$) is a good prospect for a measurement [2] of T-violation (CP is violated as well if T is violated). The reason for this is that the Standard Model (SM) contribution to this observable is very suppressed. Since $P_\mu^\perp \sim \mathcal{O}(10^{-6})$ [3] arises from electromagnetic radiative corrections in the SM, a measurement of a larger value would signal a violation of T of non-standard origin.

A preliminary measurement of the transverse polarization of the muon indicates $P_\mu^\perp = (-1.85 \pm 3.6) \times 10^{-3}$ [4], which is consistent with zero. Thus, a confirmation of $P_\mu^\perp \sim \mathcal{O}(10^{-3})$ would be an interesting tool to test physics beyond the standard model. For instance, planned experiments at KEK [2] would be sensitive to measurements of P_μ^\perp at the 5×10^{-4} level.

The calculation of this observable has been done in models containing leptoquarks [5], three doublets of Higgses [6], and tensor interactions [7]. In this brief report, which should be regarded as a complement to our previous work [8], we consider the calculation of P_μ^\perp in the context of a general two-Higgs doublet model where spontaneous violation of CP is allowed. In Ref. [8] we have considered only CP-conserving quantities in order to constrain the absolute values of the Yukawa couplings of the charged Higgs boson in the model. In the present work we consider the calculation of P_μ^\perp which would provide a bound on the phase δ that violates CP (see below). This would complete the set of constraints on the additional parameters of the model.

The part of the model relevant for our calculation is the Lagrangian for the Yukawa interactions of the charged Higgs boson, namely (see Ref. [8] for details):

$$\begin{aligned}
L_{f_i \bar{f}_j H^\pm} &= \frac{g}{\sqrt{2}m_W} H^+ \bar{U} [\cot \beta V_L^+ M_d R + \tan \beta V_L M_u L \\
&\quad + \xi e^{-i\delta_1} M_1 \Gamma L + \xi e^{-i\delta_2} M_2 \Gamma' R] D
\end{aligned}$$

$$+H^+\bar{\ell}_i[\cot\beta M_\ell\delta_{ij}R + \xi e^{-i\delta_1}M_1\Gamma_{ij}^\ell L]\nu_j + h.c., \quad (1)$$

where V_L is the Cabibbo-Kobayashi-Maskawa matrix; Γ, Γ' are dimensionless 3×3 matrices characterizing the Yukawa couplings. The small parameter ξ parametrizes the breaking of the discrete symmetry of the Lagrangian under the $\Phi_1 \rightarrow \Phi_1, \Phi_2 \rightarrow -\Phi_2$ transformations and δ_1 and δ_2 are the phases that signal CP-violation in the up- and down-type quark sectors. $M_{1,2}$, are mass parameters of the order of the W boson mass, $\tan\beta \equiv v_2/v_1$ is the ratio of v.e.v.'s for the two Higgs doublets, and finally g denotes the SU(2) coupling constant.

Since the SM contribution to CP violation and FCNC are very suppressed for the up-type quark sector, we can make a further simplification [8] (namely, $\Gamma' = 0$) in order to enhance the effects of the charged Higgs boson to the CP violation and FCNC in the up sector of quarks (in the following we use $\delta_1 = \delta$). Furthermore, since Γ is not *a priori* suppressed by the CKM non-diagonal entries one could expect that Γ gives the main contribution of charged Higgses to processes involving light flavors. As in Ref. [8] here we will neglect the contributions proportional to M_u, M_d , and M_ℓ in Eq.(1).

The amplitude for $K^+(p_K) \rightarrow \pi^0(p_\pi) + \mu^+(p_\mu) + \nu(p_\nu)$ process contains two pieces (at tree level):

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM} + \mathcal{M}_H|^2 = |\mathcal{M}_{SM}|^2 + |\mathcal{M}_H|^2 + 2\Re(\mathcal{M}_{SM}^*\mathcal{M}_H), \quad (2)$$

where the standard model contribution is

$$\mathcal{M}_{SM} = \frac{G_F}{\sqrt{2}}V_{us}\langle\pi^0|\bar{s}\gamma_\mu u|K^+\rangle\bar{u}(p_\mu, s_\mu)\gamma_\mu(1+\gamma_5)v(p_\nu), \quad (3)$$

and the charged Higgs boson contribution is given by:

$$\mathcal{M}_H = \frac{G_F}{\sqrt{2}}\lambda_{us}\lambda_{\mu\nu}e^{2i\delta}\langle\pi^0|\bar{s}u|K^+\rangle\bar{u}(p_\mu, s_\mu)(1+\gamma_5)v(q), \quad (4)$$

Here $\lambda_{ij} = \xi M_1 \Gamma_{ij} / m_H$ are dimensionless effective Yukawa couplings which values were constrained in Ref. [8].

Following Garisto and Kane in Ref. [1] the transverse muon polarization is given by

$$P_\mu^\perp = \frac{|\mathcal{M}^+|^2 - |\mathcal{M}^-|^2}{|\mathcal{M}^+|^2 + |\mathcal{M}^-|^2} \simeq \frac{4\Re(\mathcal{M}_{SM}^* \mathcal{M}_H)}{\sum_{spins} |\mathcal{M}_{SM}|^2}. \quad (5)$$

The superscripts (\pm) refer to the up and down directions of the muon spin (s_μ) respect to the decay plane.

The latter expression in Eq.(5) is obtained from the interference between the SM and scalar contributions which is proportional to the muon spin, and using the approximation $|\mathcal{M}_{SM}|^2 \gg |\mathcal{M}_H|^2$ in the denominator.

The numerator in Eq.(5) is given by

$$4\Re(\mathcal{M}_{SM}^* \mathcal{M}_H) \simeq 4\sqrt{2}G_F V_{us} \left[\frac{M_K}{m_s} \right] \frac{f_+^2}{M_W^2} [M_K \epsilon_{\alpha\beta\gamma\delta} s^\alpha p_K^\beta p_\mu^\gamma p_\nu^\delta] \mathcal{I}m\xi, \quad (6)$$

where

$$\mathcal{I}m\xi = \frac{4G_F}{\sqrt{2}} M_W^2 \lambda_{us} \lambda_{\mu\nu} \sin 2\delta. \quad (7)$$

The denominator of Eq.(5) is given by

$$\sum_{Spins} |\mathcal{M}_{SM}|^2 = 4^2 G_F^2 V_{us}^2 f_+^2 \Phi \quad (8)$$

where Φ is the phase space factor

$$\Phi = 2(p_\mu \cdot p_K)(p_\nu \cdot p_K) - M_K^2 p_\mu \cdot p_\nu + m_\mu^2 (-p_\nu \cdot p_K + \frac{p_\mu \cdot p_\nu}{4}), \quad (9)$$

and f_+ is defined from $\langle \pi^0 | \bar{s} \gamma_\mu u | K^+ \rangle \simeq f_+ (p_K + p_\pi)_\mu$.

Putting Eq.(6) and Eq.(8) into Eq.(5) we obtain (see Garisto and Kane in Ref. [1])

$$P_\mu^\perp \simeq \frac{\sqrt{2}}{4} (G_F M_W^2 V_{us})^{-1} \left[\frac{M_K}{m_s} \right] \left[\frac{M_K \epsilon_{\alpha\beta\gamma\delta} s^\alpha p_K^\beta p_\mu^\gamma p_\nu^\delta}{\Phi} \right] \mathcal{I} m \xi. \quad (10)$$

As pointed out in Ref. [9], it is convenient to define an average value for P_μ^\perp . We define the average value for P_μ^\perp in the K^+ rest frame as follows:

$$\begin{aligned} \overline{P_\mu^\perp} &= \int P_\mu^\perp dp d\theta \\ &\simeq 7.06 \left(\frac{0.199 \text{ GeV}}{m_s} \right) \lambda_{us} \lambda_{\mu\nu} \sin 2\delta, \end{aligned} \quad (11)$$

where the two independent kinematical variables are taken as $p \equiv \frac{|\vec{p}|}{M_K}$ and the angle θ ($\cos\theta \equiv \hat{p}_\mu \cdot \hat{p}_\nu$). In the above numerical result we have neglected terms of order m_μ^2/M_K^2 in the expression for Φ (as done in Garisto and Kane, Ref. [1]).

As expected, the transverse muon polarization is proportional to δ , the CP-violating phase that appears in the Yukawa couplings in Eq.(1). Thus a measurement of P_μ^\perp would provide the value of $\lambda_{us} \lambda_{\mu\nu} \sin 2\delta$.

In order to get a conclusion on the phase δ , we can proceed as follows. As is pointed in Ref. [10] (p.p. 1530-1531), if we relax the V-A requirement for the weak charged current responsible for the $K^+ \rightarrow \pi^0 e^+ \nu_e$ process, we can allow in particular an scalar contribution of the form

$$\mathcal{M}_S = \frac{G_F}{\sqrt{2}} V_{us} (2M_K) f_S \bar{\ell} (1 + \gamma_5) \nu_\ell. \quad (12)$$

If we attribute this amplitude to the exchange of the charged Higgs of our model, Eq.(12) becomes identical to Eq.(4). This would imply:

$$|\lambda_{us}\lambda_{e\nu}| \simeq 2M_K V_{us} \left(\frac{m_s - m_u}{M_K^2 - m_\pi^2} \right) \left| \frac{f_S}{f_+} \right|. \quad (13)$$

If we take the experimental value reported in Ref. [11], namely $|\frac{f_S}{f_+}| = 0.084 \pm 0.023$ and if we assume $e - \mu$ universality, we would obtain:

$$|\lambda_{us}\lambda_{\mu\nu_\mu}| \simeq (1.6 \times 10^{-2}) \left(\frac{m_s}{0.199 \text{ GeV}} \right), \quad (14)$$

which is consistent with the upper bound obtained in Ref. [8]. Using Eq.(14) in Eq.(11), we get

$$\overline{P}_\mu^\perp \simeq (0.11) \sin 2\delta. \quad (15)$$

Thus, a measurement of \overline{P}_μ^\perp of the order of $10^{-3} - 10^{-4}$ would imply a small phase, $\delta \sim \mathcal{O}(10^{-2} - 10^{-3})$. In fact, since $\sin 2\delta \leq 1$, we expect $\overline{P}_\mu^\perp \leq 10^{-1}$.

Summarizing, we have calculated the transverse muon polarization in the framework of a general two-Higgs doublet model with spontaneous violation of CP. Using present bounds on the absolute value of the effective Yukawa couplings of the charged Higgs boson (Ref.[8] and Eq.(14)), we estimate $\overline{P}_\mu^\perp \simeq 0.11 \sin 2\delta$. Thus, a measurement of \overline{P}_μ^\perp at the level of $10^{-3} - 10^{-4}$, would imply $\delta \sim \mathcal{O}(10^{-2} - 10^{-3})$. We would like to emphasize that \overline{P}_μ^\perp can not be induced from a scalar amplitude arising from two-Higgs doublet model where we impose the symmetry under $\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$ (namely $\xi = 0$ in Eq.(1)).

Acknowledgements

I would like to thank G. López Castro for suggesting this problem and for useful discussions.

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